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A Precision, High Frequency Calibration Facility for Coaxial Capacitance Standards

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A PRECISION, HIGH FREQUENCY CALIBRATION FACILITY FOR COAXIAL CAPACITANCE STANDARDS

By

R. N. Jones and L. E. Huntley

ABSTRACT

Using high frequency impedance standards which are fitted with precision coaxial connectors, and limiting calibrations to specific frequencies and impedance values, can contribute greatly toward improving measurement agreement and standardization. These advantages have been utilized in the development of a new and much improved calibration service for capacitance in the high frequency region. The service is for capacitances of 50, 100, 200, 500 and 1000 picofarads at 100 kHz, 1 MHz, and 10 MHz. This paper describes the instrumentation used in the measurements and the method of data handling, and gives a detailed breakdown of the measurement uncertainties.

Key words: Calibration; Capacitance; Capacitance, Measurement; Coaxial Capacitance Standards; Impedance, Measurements, Capacitance; Standards; Statistical Control.

I. INTRODUCTION

It has been the practice of NBS to provide two-terminal impedance calibrations at any frequency or any impedance value which its clients have requested in the high frequency region (30 kHz to 300 MHz). In accordance with this general practice, there have been virtually no restraints imposed as to the types of impedance standards accepted for calibration. The only requirements have been that the device to be calibrated be stable to the order of about ± 0.01 percent and that the calibration frequency be far enough away from the self resonant frequency of the device that uncertainties in measurement frequency do not give rise to errors greater than about 0.01 percent.

Such a practice has been a necessity because of the wide variation in the types of impedance standards being used throughout the country. In many situations, clients have manufactured their own standards because appropriate commercial standards were unavailable. A second reason that standards vary so much has been the nonuniformity in connectors. For example, NBS has performed calibrations of standards equipped with banana plugs or binding posts with spacings which vary from 5/8" to 2 1/4". This, of course, is indicative of the variation which exists between various commercial instruments and presents some difficult problems in correlating measurement data between instruments with dissimilar connectors.

Providing calibration services for such a variety of standards, while it offers convenience to the client, carries with it some serious disadvantages. The major disadvantages are cost and difficulty in assessing measurement uncertainties. Higher calibration costs result when special adaptation methods must be employed to carry out a measurement procedure. It is not infrequent that such costs exceed what the customer is able to afford and calibrations are either sharply cut back or left undone. In addition to higher costs, there is the problem of being unable to perform a comprehensive assessment of measurement uncertainties. Thus, unusual calibrations are more expensive and less accurate than those where uniform standards and calibration frequencies are employed.

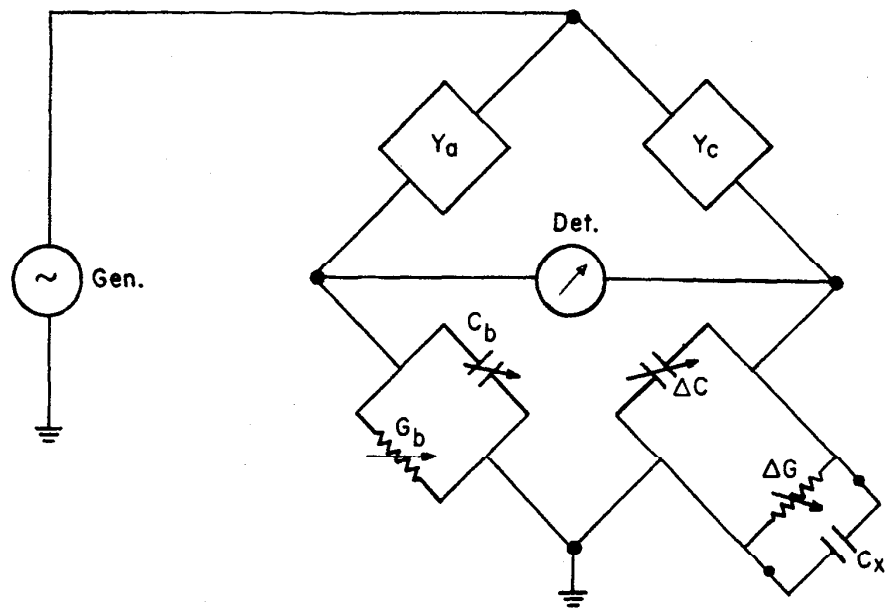
This NBS Technical Note describes a pilot procedure which offers an alternative to the more expensive and less accurate results obtained in calibrating nonuniform standards. As standards equipped with precision coaxial connectors^[1] became available commercially, it became feasible to establish calibration services which utilize statistical data to evaluate uncertainty limits. Employing uniform standards, and limiting calibrations to specific impedance values and frequencies, allows the accumulation of the data required for statistical treatment.

The standards and calibration services to be described in this report are the first in the area of high frequency impedance to be treated by the statistical approach to this extent. The service is for two-terminal, air-dielectric capacitors having values of 50, 100, 200, 500, and 1000 picofarads. To qualify for this service, a capacitor must be equipped with a 14 mm precision coaxial connector, and be within ± 1 picofarad of its nominal value when measured at 1 kHz. Calibration frequencies are 0.1, 1.0, and 10 MHz. The determination of conductance or dissipation factor is not included as part of the calibration because these quantities are extremely small in capacitors of this type and accurate determination at these frequencies is beyond the present state-of-the-art.

[1] IEEE Standard for Precision Coaxial Connectors, IEEE Trans. Instr. Meas., Vol. IM-17, No. 3, pp. 204-218, Sept. 1968. Also published as Standard Publication No. 287, available from the Institute of Electrical and Electronics Engineers, Inc., 345 East 47 Street, New York, New York 10017.

II. INSTRUMENTATION AND MEASUREMENT TECHNIQUE

For each of the nominal values mentioned in I., there are three NBS working standards. In the calibration procedure, the difference between the unknown and each of the three NBS standards is determined. This is accomplished by means of an admittance ratio bridge (fig. 1), which is used in this application as a simple comparator. ΔC is a coaxial incremental capacitor with a range of two picofarads and a resolution of 0.0001 picofarad. Balance is initially established with any one of the three NBS standards or the unknown connected at the location indicated by C_x in fig. 1. Successive readings of ΔC are then obtained by balancing the bridge with the other standards, and the unknown, located at position C_x . A photograph of the comparator is shown in fig. 2. The adjustable components are attached to the comparator externally and are labeled in the photograph. The ratio admittances Y_a and Y_c are mica capacitors located in the base of the instrument. These mica capacitors are easily removable so that the condition for maximum sensitivity (i.e., all bridge arms having equal impedance) may be provided for each unknown being calibrated. Throughout the calibration procedure, ΔC is the only item to which any adjustment is made.



Y_a and Y_c - Ratio immittances

$$\frac{Y_a}{Y_c} \approx 1$$

C_b - Capacitor for initial bridge balance

G_b - Conductance for initial bridge balance

ΔC - Incremental capacitor (5 - 7 pF)

ΔG - Variable conductance

C_x - Standard or unknown

Fig. 1 Immittance Comparator Circuit Diagram

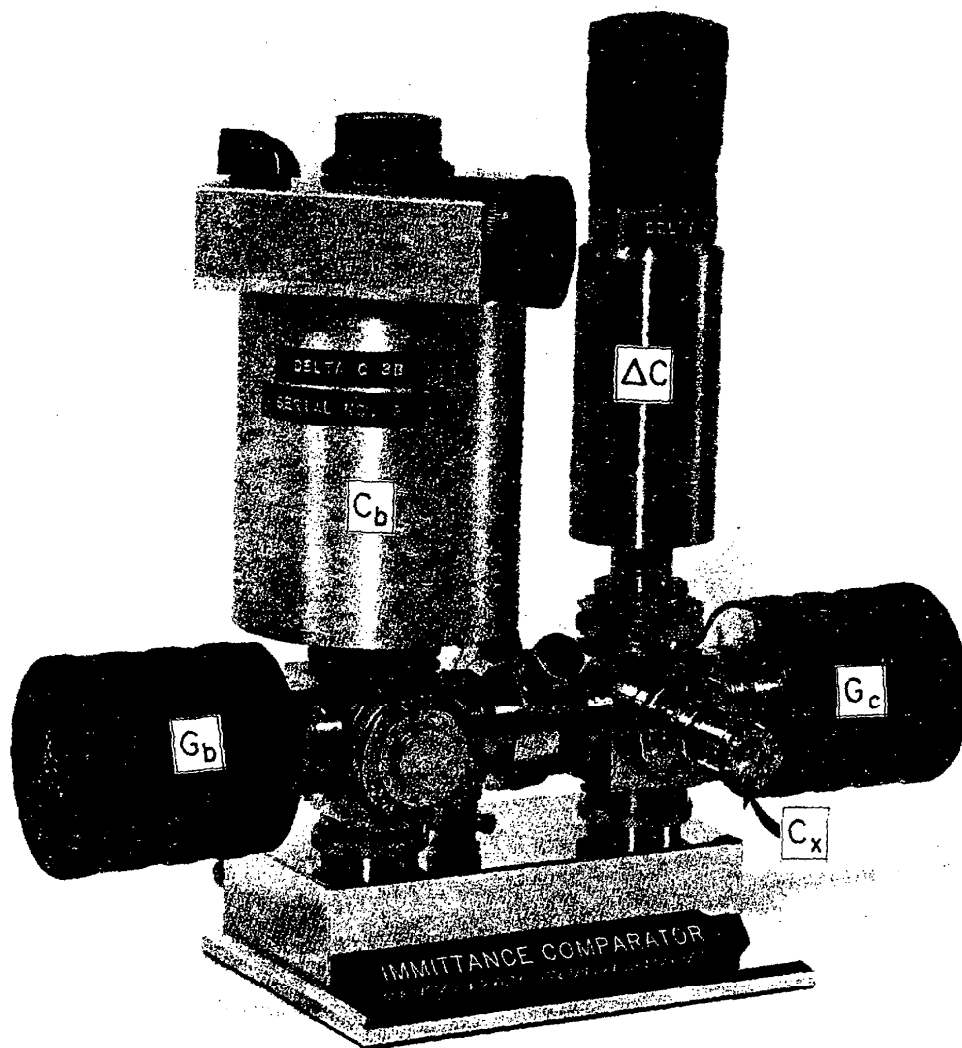


Fig. 2 Immittance Comparator

In order to achieve a true measurement of the relative differences between the standards and the unknown, the balance condition of the comparator must be dependent upon the setting of ΔC only. This means that the conductances, G_b and ΔG , and the capacitance C_b , must remain at their original settings throughout the procedure. The reason is obvious in the case of C_b , because a change in C_b would change the reference balance condition of the comparator. G_b and ΔG must remain at their original settings because these components have residual reactances which may vary with setting and result in an error in the capacitance difference measurements. The requirement that the capacitance balance be dependent only on the setting of ΔC presents a problem that is solved by the use of a detector system which permits the separation of the bridge output signal into its in-phase and quadrature components.

The detection system employs a two-phase, lock-in amplifier wherein the output signal from the bridge is compared with a reference signal of the same frequency.^[2] Both signals are derived from the bridge oscillator. The heart of the lock-in amplifier is a phase sensitive detector which permits the operator to differentiate between bridge unbalance due

[2] R. D. Moore, "Lock-in Amplifier for Signals Buried in Noise," Electronics, June 8, 1962, McGraw-Hill Publishing Co., New York, New York.

to conductance as opposed to that due to capacitance. This eliminates the need to adjust the bridge conductance standard in order to achieve a good capacitance balance. The detector scheme is shown in fig. 3. At present, lock-in amplifiers are available which operate up to only about 0.1 MHz. To permit the use of this method of detection up to 10 MHz, a dual-channel converter containing a local oscillator is used to beat the reference and bridge output signals down to a frequency within the range of the lock-in amplifier. This detector system is capable of very high sensitivity and enables the capacitance differences to be resolved to ± 0.0001 picofarad over the 0.1 to 10 MHz range.

III. STATISTICAL APPROACH TO DATA HANDLING

In the measurement procedure described in the previous section, let us assume that we initially connect NBS standard No. 1 and obtain a reading C_{11} on the incremental capacitor, ΔC . Successive readings for ΔC are then obtained with NBS standard No. 2, NBS standard No. 3 and the unknown capacitor located at position C_x (fig. 1). These operations produce balance readings of C_{11} , C_{12} , C_{13} and C_{1x} respectively. By these four measurements we can establish the differences in the capacitance of the four capacitors, each relative to the other three. The process just described is repeated n times (n is usually 6). The data (ΔC readings from each of the n

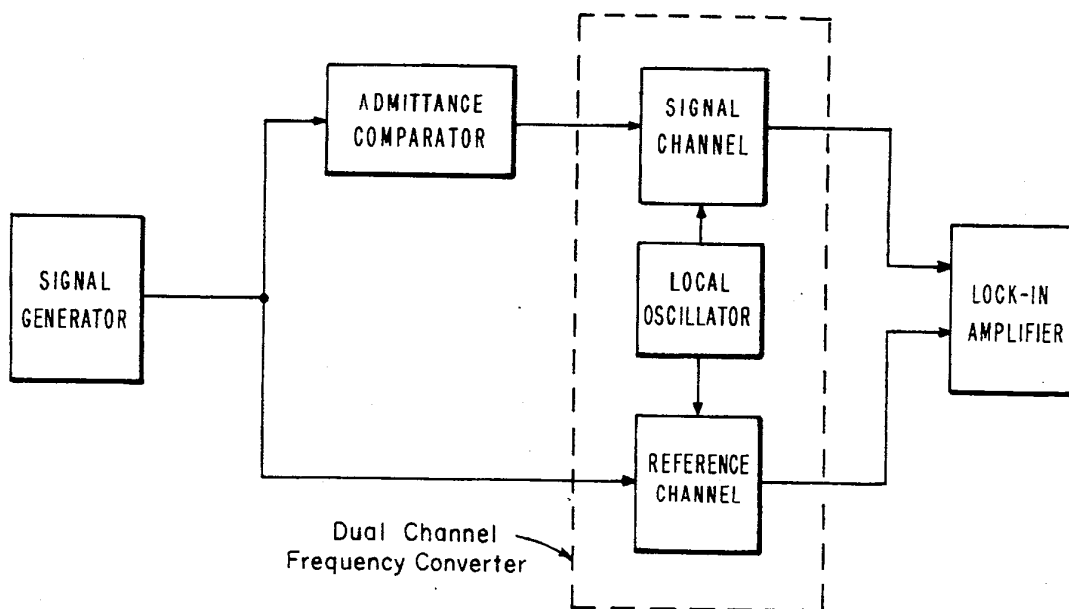


Fig. 3 Block Diagram of Calibration Setup

runs) may be arranged as follows:

C_{11}	C_{12}	C_{13}	C_{1x}
C_{21}	C_{22}	C_{23}	C_{2x}
C_{31}	C_{32}	C_{33}	C_{3x}
.	.	.	.
.	.	.	.
.	.	.	.
C_{n1}	C_{n2}	C_{n3}	C_{nx}

A statistical procedure is used to obtain the reported calibration results and also to obtain control chart information. Let us proceed to the explanation of this statistical procedure by first considering the following diagram. (fig. 4)

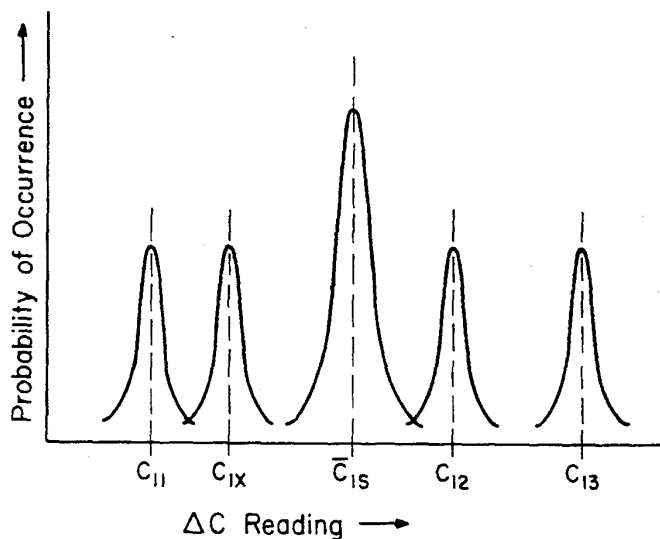


Fig. 4 Conceptual Statistical Representation of Calibration Data

Here C_{11} , C_{12} , C_{13} and C_{1x} are the ΔC readings, for the three NBS standards and the unknown for the first run. The

curves of figure 4 suggest that each value falls within some distribution which we will assume for the moment to be normal, or Gaussian.

For each run of data an average C reading is obtained for the NBS standards

$$C_{is} = (C_{i1} + C_{i2} + C_{i3})/3; i=1 \dots n. \quad (1)$$

Using the differences:

$$D_{i1} = C_{i1} - C_{is}, \quad (2)$$

$$D_{i2} = C_{i2} - C_{is}, \quad (3)$$

$$D_{i3} = C_{i3} - C_{is}, \quad (4)$$

obtained from each of the n runs of data, average differences are computed:

$$\bar{D}_1 = \frac{1}{n} \sum_{i=1}^n D_{i1}, \quad (5)$$

$$\bar{D}_2 = \frac{1}{n} \sum_{i=1}^n D_{i2}, \quad (6)$$

$$\bar{D}_3 = \frac{1}{n} \sum_{i=1}^n D_{i3}. \quad (7)$$

One reason for using differences between observed readings, instead of just the observed readings themselves, is to reduce the effects of any drift or change in the bridge network. A second reason is that in this part of the procedure the measurements are used to determine the values of the standards, each relative to the others, as opposed to making

absolute measurements of individual capacitors. The estimated variances about these average differences are given by:

$$V_1 = s_1^2 = \frac{1}{n-1} \sum_{i=1}^n (D_{i1} - \bar{D}_1)^2, \quad (8)$$

$$V_2 = s_2^2 = \frac{1}{n-1} \sum_{i=1}^n (D_{i2} - \bar{D}_2)^2, \quad (9)$$

$$V_3 = s_3^2 = \frac{1}{n-1} \sum_{i=1}^n (D_{i3} - \bar{D}_3)^2. \quad (10)$$

In addition to the average differences \bar{D}_1 , \bar{D}_2 , and \bar{D}_3 obtained in (5), (6), and (7), we also compute average differences which include all previous runs m in number. These are called \bar{D}'_1 , \bar{D}'_2 , and \bar{D}'_3 where:

$$\bar{D}'_1 = \frac{1}{m} \sum_{i=1}^m D_{i1}, \quad (11)$$

$$\bar{D}'_2 = \frac{1}{m} \sum_{i=1}^m D_{i2}, \quad (12)$$

$$\bar{D}'_3 = \frac{1}{m} \sum_{i=1}^m D_{i3}. \quad (13)$$

The variances about \bar{D}'_1 , \bar{D}'_2 , and \bar{D}'_3 are also computed:

$$V'_1 = s'^2_1 = \frac{1}{m-1} \sum_{i=1}^m (D_{i1} - \bar{D}'_1)^2, \quad (14)$$

$$V'_2 = s'^2_2 = \frac{1}{m-1} \sum_{i=1}^m (D_{i2} - \bar{D}'_2)^2, \quad (15)$$

$$V'_3 = s'^2_3 = \frac{1}{m-1} \sum_{i=1}^m (D_{i3} - \bar{D}'_3)^2. \quad (16)$$

In the expressions (11) through (16), m does not include the n runs of the most recent series of measurements.

Using the results of the foregoing computations, control charts are constructed, and updated after each series of comparisons. [3] Control charts aid in determining whether or not the measurement system and standards are behaving in a predictable manner. When this is the case, the measurements are said to be "in control," and well-justified bounds of measurement uncertainty may be determined.

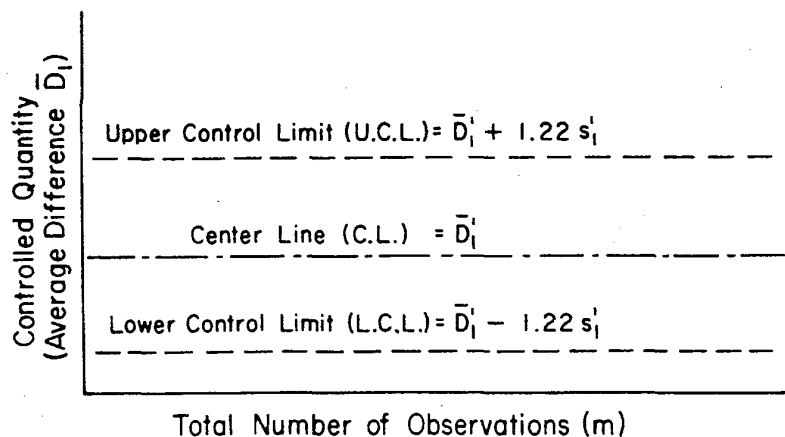


Fig. 5 Control Chart for Average Differences

Figures 5 and 6 are indicative of the types of control charts in use. In fig. 5 the average difference \bar{D}_1 is

[3] M. G. Natrella, "Experimental Statistics," NBS Handbook 91, August 1, 1963.

plotted each time a new run of 6 observations is obtained. The center line on the control chart is the grand mean \bar{D}_1' of all the previous average differences \bar{D}_1 . Upper and lower control limits are based upon the standard deviation s_1' in the average differences $(\bar{D}_1 - \bar{D}_1')$ of all previous runs. The factor 1.22 comes from evaluating the expression $\frac{k}{\sqrt{n}}$ where k is the factor by which the standard deviation is multiplied to give a particular confidence level, and n is the number of observations. In this instance $k = 3$ for 99.7% confidence level and $n = 6$. In fig. 6 the controlled quantity is the standard deviation of the average differences and, as in fig. 5, the upper and lower control limits are based upon s_1' , with the multiplier now being $\frac{k}{\sqrt{2(n-1)}} = 0.948$. The factor c used in connection with the center line is a correction term which is used to allow for the fact that values of s_1 are biased estimates of the value for the population standard deviation. For $n = 6$, $c = 0.952$.

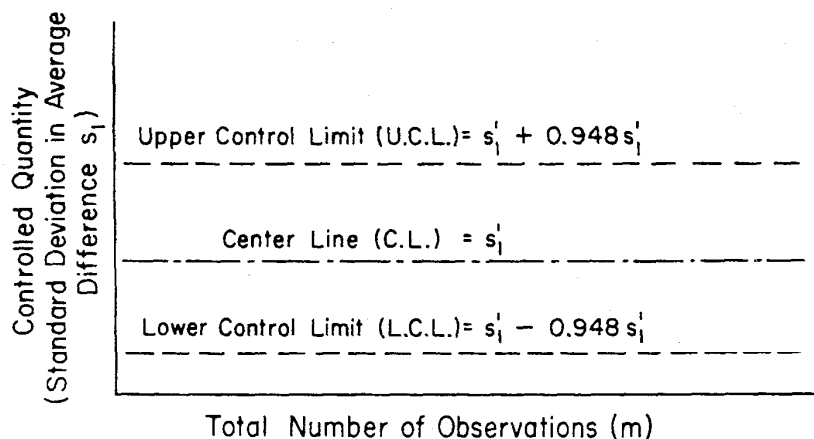


Fig. 6 Control Chart for Standard Deviations in Average Differences

With the center line and upper and lower control limits determined, the results of each set of six determinations are plotted to determine whether or not they fall within the predicted control limits. Because the limiting mean and standard deviations in the average difference are initially unknown, the center line and control limits may require adjustment until sufficient data is obtained to arrive at stable values based on a large number of observations over a long time interval.

Section IV. contains a discussion of the absolute values of the three NBS working standards and how they were obtained but, for the moment, assume that the absolute values are known at a particular frequency and call them C_{A1} , C_{A2} and C_{A3} . In this calibration process, the standard, C_s , is the average of the absolute values of the three capacitors or

$$C_s = \frac{C_{A1} + C_{A2} + C_{A3}}{3}. \quad (17)$$

Thus the standard is embodied by the three individual NBS working standards.

Up to this point nothing has been said about the unknown capacitor being calibrated. The next step is to obtain relationships between the unknown and the NBS standards, thereby permitting the determination of the value of the unknown.

For this purpose define

$$B_i = C_{ix} - C_{is}; i=1 \dots n \quad (18)$$

and

$$\bar{B} = \frac{1}{n} \sum_{i=1}^n B_i. \quad (19)$$

The value of the unknown capacitor, C_x , is then given by:

$$C_x = C_s + \bar{B} \quad (20)$$

and the variance about this value is

$$V_x = s_x^2 = \frac{1}{n-1} \sum_{i=1}^n (B_i - \bar{B})^2. \quad (21)$$

\bar{B} , of itself, is either positive or negative depending upon whether the unknown capacitor is greater or less than the average of the three NBS working standards.

Because of the relatively small sample of data obtained for each unknown capacitor, ($n=6$), we have used the student's-t distribution in determining uncertainty limits. Therefore the standard deviation, s_x , is multiplied by 1.65* to arrive at a value of the 99 percent confidence level. The term $1.65s_x$ is included as a part of the "Comparison Uncertainty" in the Report of Calibration.

* Student's t for 99 percent confidence level and 5 degrees of freedom is 4.03. Dividing this by $\sqrt{6}$ gives 1.65 as the multiplier for s_x to yield confidence limits for the mean of 6 measurements.

IV. DETERMINATION OF ABSOLUTE VALUES OF NBS WORKING STANDARDS

Having described the comparison procedure and statistical operations associated with the maintenance of the NBS working standards, there remains to be described the technique used in determining the absolute values of the NBS working standards.

The process begins with a measurement of the absolute value of the capacitor at 1 kHz. This is done by means of a transformer ratio arm admittance bridge^[4] and a special adaptor which enables absolute two-terminal capacitance measurements to be made on a three-terminal bridge.*

The high frequency value, C_e , of an NBS working standard is calculated from the expression

$$C_e = \frac{C_o}{1 - \omega^2 L_r C_o} \quad (22)$$

where ω = the angular frequency in radians per second

C_o = the capacitance in farads measured at 1 kHz.

L_r = the residual inductance of the capacitor in henries.^[5]

[4] M. C. McGregor, J. F. Hersh, R. D. Cutkosky, F. K. Harris and F. R. Kotter, "New Apparatus at the National Bureau of Standards for Absolute Capacitance Measurement," I.R.E. Trans. on Inst., Vol. I-7, Nos. 3 & 4, Dec. 1958, pp. 253-261.

* See appendix (B) for a description of the two-terminal to three-terminal adaptor.

[5] R. N. Jones, "A Technique for Extrapolating the 1 kc Values of Secondary Capacitance Standards to Higher Frequencies," NBS Tech. Note No. 201, November 5, 1963.

Reference [5] is used here to direct the reader to a resonance method used to measure the residual inductance of a capacitor. The reference also contains a discussion of some of the uncertainties involved. In the present situation, the residual inductance determination was performed in an all coaxial circuit configuration using shorted sections of precision coaxial line to resonate the capacitor. These inductance determinations were made at a number of different frequencies by resonating the capacitors with different lengths of shorted coax line and the resonant frequencies were determined to a few parts in 10^7 or better. Resonance experiments performed at a number of different frequencies provided assurance that no appreciable errors were incurred by assuming that the capacitors could be represented by a single series L-C equivalent circuit. These resonance experiments were performed over a frequency range from 10 to 100 MHz and in no case did the values obtained for residual series inductance vary by more than ± 10 percent. Agreement of this order has also been achieved by an alternate method which utilizes a slotted line. [6]

[6] J. Zorzy and M. J. McKee, "Precision Capacitance Measurements with a Slotted Line," General Radio Experimenter, Vol. 41, No. 9, pp. 10-11, September 1967.

V. ADAPTOR CORRECTION

In the actual measurement situation, it is necessary to utilize a coaxial adaptor between the unknown terminal of the immittance comparator and a standard or unknown capacitor equipped with a 14 mm precision coaxial connector. This introduces a systematic error or bias in the difference measurements which is a function of the error in the value used for the inductance of the adaptor, and the measurement frequency. This bias which is denoted as B is given by:

$$*B = \Delta C(1 + 2\omega^2 L_A C) \text{ where } \Delta C = \text{capacitance difference between unknown and standard at 1 kHz.}$$

ω = angular frequency of measurement.

L_A = inductance in series with unknown or standard.

$C = C_x \approx C_s$ = Absolute capacitance of unknown or standard.

The adaptor used for these calibrations has a value for $L_A \approx 15 \times 10^{-9}$ henries. This value is used in evaluating B, but at these calibration frequencies, there is a wide range of uncertainty and actual measurement is extremely difficult.

* See appendix (A) for derivation.

In the estimates of systematic error a 100 percent uncertainty was allowed in L_A . For the worst case, which is the calibration of a 1000 pf capacitor at 10 MHz, the uncertainty due to B would be equal to 0.12% of C, if $\Delta C = 1$ pf.

VI. CALIBRATION UNCERTAINTIES

The method of reporting the uncertainties associated with these calibrations is based upon the premise that repeatability or measurement agreement throughout the nation is often of greater importance than absolute accuracy. While it is desirable to have both precision and accuracy, the fact remains that it is often possible to make measurements which are much more precise than they are accurate. With this in mind the uncertainty statement has been separated into two categories; one called the "Comparison Uncertainty" and the other called "Uncertainty in Absolute Value of NBS Standard." With this approach, measurement agreement need not be limited by the state of the art as far as knowledge of the absolute value of an item or measurand is known.

Table 1 is a tabulation of the magnitudes of uncertainties from five sources designated a, b, c, d and e. In addition the tabulation separates the errors as a function of frequency and nominal value of the capacitor calibrated. Factors a and b are added to give "Uncertainty in Absolute Value of NBS Standard." Factors c, d and e are added to give the "Comparison Uncertainty." The explanation of the meaning of each of these factors is as follows:

TABLE 1.
MEASUREMENT UNCERTAINTIES

Nominal Capacitance of Unknown Capacitor at 1 kHz in picofarads	Factor	Uncertainties in (±) %			
		1 kHz	100 kHz	1 MHz	10 MHz
50	(a)	0.009			
	(b)		<0.0001	0.0002	0.02
	(c)		0.001	0.001	0.002
	(d)	0.01			
	(e)		<0.0001	<0.0001	0.006
100	(a)	0.006			
	(b)		<0.0001	0.0004	0.04
	(c)		0.002	0.002	0.003
	(d)	0.01			
	(e)		<0.0001	0.0001	0.01
200	(a)	0.004			
	(b)		<0.0001	0.0008	0.08
	(c)		0.003	0.002	0.002
	(d)	0.01			
	(e)		<0.0001	0.0002	0.02
500	(a)	0.004			
	(b)		<0.0001	0.002	0.2
	(c)		0.005	0.003	0.004
	(d)	0.01			
	(e)		<0.0001	0.0006	0.06
1000	(a)	0.004			
	(b)		<0.0001	0.004	0.4
	(c)		0.05	0.03	0.04
	(d)	0.01			
	(e)		<0.0001	0.001	0.12

- (a) The uncertainty of the value of the NBS standard as measured at 1 kHz.
- (b) The uncertainty in the high frequency value of the NBS standard resulting from a $\pm 10\%$ uncertainty in the residual inductance.
- (c) The imprecision in the intercomparison of the unknown with the NBS standard based upon 99 percent confidence limits and the Student's t distribution for a sample size, $n = 6$. See footnote at bottom of page 13.
- (d) A factor of 0.01 percent to allow for a long term drift in the NBS standard.
- (e) The uncertainty associated with the inductance of the adaptor used in the calibration setup. This is a bias uncertainty which is calculated for each calibration assuming a 100 percent uncertainty in the inductance, L_A .

VII. CONCLUSION

By the techniques described, an attempt has been made to inject a degree of sophistication into impedance measurement and standardization which have not been either practical or possible heretofore. The fact that this is now possible is primarily because precision coaxial connectors are now widely available.

This approach to calibration is dependent upon wide acceptance of specific frequencies and capacitance values around which the standards, equipment, and measurement techniques are designed. The advantages are many fold and include the ability to use a high speed digital computer for data handling, the incorporation of statistical methods in the treatment of data and improvements in accuracy and precision.

Similar endeavors in the dissemination of high frequency impedance standards could well be undertaken for other values of capacitance, for inductance, and for resistance. How soon this will be done is not certain, but it will depend upon the number of calibration requests received for the new high frequency capacitance facility.

APPENDIX A

ERROR DUE TO INDUCTANCE, L_A , IN SERIES WITH C_X OR C_S

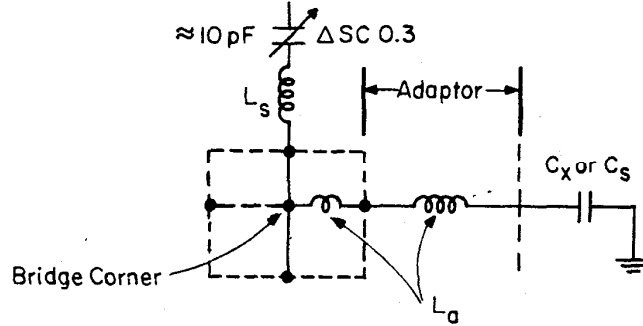


Fig. 7 Equivalent Circuit for Analysis of Error Due to Inductance in Series with Unknown

From the bridge corner looking toward C_X or C_S :

$$*C_{e_x} = C_x(1 + \omega^2 L_A C_x) = C_x + \omega^2 L_A C_x^2$$

$$C_{e_s} = C_s(1 + \omega^2 L_A C_s) = C_s + \omega^2 L_A C_s^2$$

$$\Delta C_e = C_{e_x} - C_{e_s} = C_x + \omega^2 L_A C_x^2 - C_s - \omega^2 L_A C_s^2$$

$$\Delta C_e = C_x - C_s + \omega^2 L_A (C_x^2 - C_s^2)$$

$$\Delta C_e = C_x - C_s + \omega^2 L_A (C_x - C_s)(C_x + C_s)$$

$$\text{let } \Delta C = (C_x - C_s)$$

$$\Delta C_e = \Delta C + \omega^2 L_A (C_x + C_s) \Delta C$$

$$\text{for } C_x \approx C_s$$

$$\Delta C_e = \Delta C(1 + 2\omega^2 L_A C_s) \text{ which is denoted as B in the text.}$$

The error due to L_s in series with $\Delta SC0.3$ is negligible because the capacitance is only about 10 pF.

$$*C_e = \frac{C_o}{1 - \omega^2 L C_o}; \text{ but } C_e \approx C_o + \omega^2 L C_o^2 \text{ at frequencies well below resonance.}$$

APPENDIX B

ABSOLUTE TWO-TERMINAL CAPACITANCE MEASUREMENTS

USING A TRANSFORMER RATIO ARM BRIDGE

The most accurate capacitance measurements which it is possible to make, at the present state of the art, are performed using a transformer ratio arm bridge.^[4] These are usually three-terminal capacitance measurements and are performed at a frequency of 1 kHz. When a good two-terminal capacitor is equipped with a precision coaxial connector, it can be evaluated to uncertainties of the order of ten parts per million using a three-terminal to two-terminal coaxial adaptor and a transformer ratio arm bridge. This technique is used extensively in the high frequency impedance laboratory at NBS Boulder.

To understand the technique, examine first the adaptor which must be used. The diagram of the adaptor is shown in fig. 8.

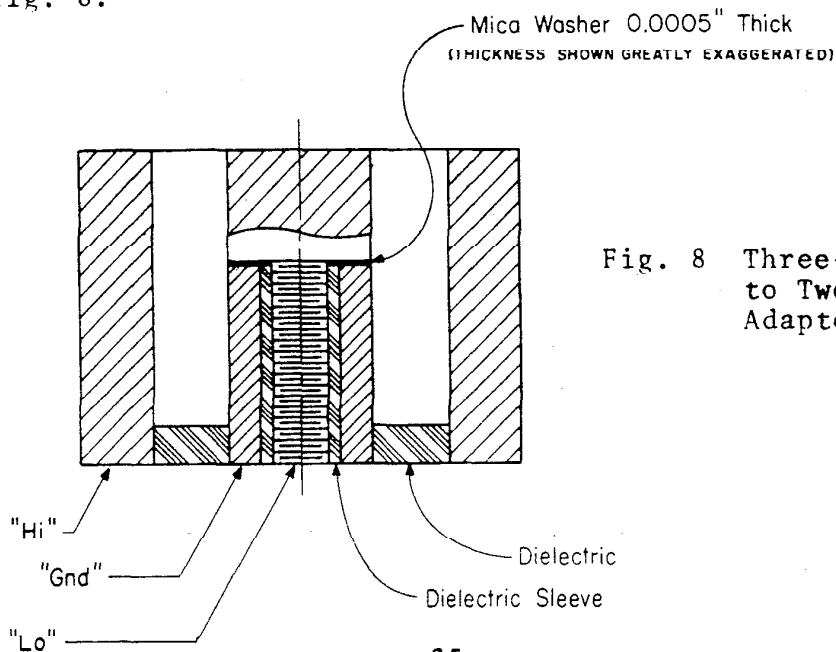


Fig. 8 Three-Terminal to Two-Terminal Adaptor

A 0.0005" thick mica washer and a dielectric sleeve, insulate the two parts of the center conductor from each other, and the center conductor is also insulated from the outer conductor. The adaptor is connected to the transformer ratio bridge as indicated by fig. 9.

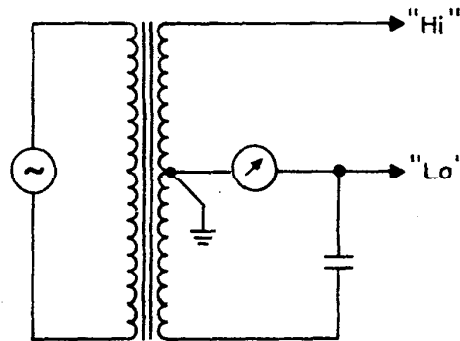


Fig. 9 Transformer Ratio-Arm Bridge

The bridge measures only the direct capacitance which is connected between the "Hi" and "Lo" terminals, thus the effective capacitance of the adaptor is only that amount of adaptor capacitance appearing between the top section of the center conductor and the outer conductor (fig. 8). Because this is a uniform coaxial section, we can calculate its capacitance very accurately if we know the length of the section, and the diameters of the inner and outer conductors. The presence of the thin mica washer, and the fact that the ground and "Lo" terminals come to the same potential at bridge balance, assure that there will be no electric

field across the gap occupied by the thin mica washer, and a practically uniform field from the inner conductors to the outer conductor in the vicinity of the gap. This, in turn, provides assurance that the capacitance measured by the bridge is identical to that calculated for the outer coaxial section of the adaptor. By this means absolute capacitance measurements can be made with respect to the top surface of the adaptor by merely subtracting the adaptor capacitance from the bridge reading.